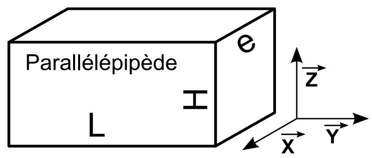
TD – Matrices d'inertie / Palpeur MMT

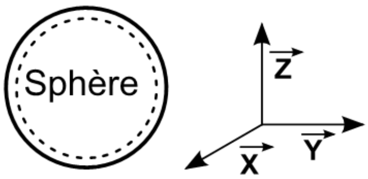
POINT METHODE :

- Forme des matrices d'inertie (**Q1**) :



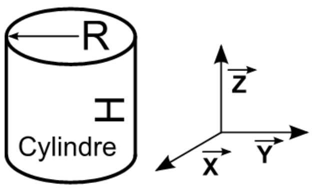
Deux Plans de symétrie $(Q,\vec{x},\vec{y})$ et $(Q,\vec{x},\vec{z})$:

D=0 et E=0 et F=0 → $\begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}_{Qxyz}$



Un centre de symétrie sphérique (G forcément) :

A=B=C et D=0 et E=0 et F=0 et A+B+C=3A → $\begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix}_{Q}$



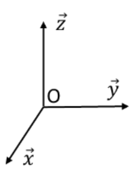
Un axe de Révolution $(Q,\vec{z})$:

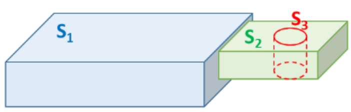
A = B et D=0 et E=0 et F=0 et A+B=2A → $\begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{bmatrix}_{Qz}$

- Centre de gravité d'un ensemble de solides (**Q2**) :

$$\left(\sum_{i=1}^{n} m_i\right)\overrightarrow{OG} = M\overrightarrow{OG} = \sum_{i=1}^{n} m_i\,\overrightarrow{OG_i} \quad \text{ou} \quad \sum_{i=1}^{n} m_i\,\overrightarrow{GG_i} = \vec{0}$$

- Somme de matrices d'inertie (**Q3**) :





$\Sigma = S_1 + S_2 - S_3$

$$[I_O(\Sigma)]_R = [I_O(S_1)]_R + [I_O(S_2)]_R - [I_O(S_3)]_R$$

- Théorème de Huygens (**Q3**) :

$$[I_P(S)] = [I_G(S)] + m\begin{bmatrix} b^2+c^2 & -ab & -ac \\ -ab & a^2+c^2 & -bc \\ -ac & -bc & a^2+b^2 \end{bmatrix}$$ Avec $\overrightarrow{PG} = a\,\vec{x} + b\,\vec{y} + c\,\vec{z}$

ELEMENTS DE CORRECTION :

Q1 :

$$G_1 = \begin{pmatrix} -\frac{H}{2} \\ \frac{L}{2} \\ \frac{e}{2} \end{pmatrix}_{(\vec{x},\vec{y},\vec{z})} \quad \text{et } \left[I_{G_1}(S_1)\right] = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & B_1 & 0 \\ 0 & 0 & C_1 \end{bmatrix}_{G_1,(\vec{x},\vec{y},\vec{z})}$$

$$G_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{(\vec{x},\vec{y},\vec{z})} \quad \text{et } \left[I_{G_2}(S_2)\right] = \begin{bmatrix} A_2 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_2 \end{bmatrix}_{G_2,(\vec{x},\vec{y},\vec{z})}$$

$$G_3 = \begin{pmatrix} 0 \\ 0 \\ \frac{e}{2} \end{pmatrix}_{(\vec{x},\vec{y},\vec{z})} \quad \text{et } \left[I_{G_3}(S_3)\right] = \begin{bmatrix} A_3 & 0 & 0 \\ 0 & A_3 & 0 \\ 0 & 0 & C_3 \end{bmatrix}_{G_3,(\vec{x},\vec{y},\vec{z})}$$

Q2 :

$$\overrightarrow{OG} = -\frac{H}{2}.\vec{x} + \frac{M_1.\frac{L}{2} + M_2.\left(L + \frac{D}{2}\right) - M_3.(2.a + b)}{M_1 + M_2 + M_3}.\vec{y} + \frac{e}{2}.\vec{z}$$

Q3 :

$$[I_O(S_1)] = \begin{bmatrix} A_1 + M_1.\frac{L^2 + e^2}{4} & M_1.\frac{H.L}{4} & M_1.\frac{H.e}{4} \\ M_1.\frac{H.L}{4} & B_1 + M_1.\frac{H^2 + e^2}{4} & -M_1.\frac{L.e}{4} \\ M_1.\frac{H.e}{4} & -M_1.\frac{L.e}{4} & C_1 + M_1.\frac{L^2 + H^2}{4} \end{bmatrix}_{O,(\vec{x},\vec{y},\vec{z})}$$

$$[I_O(S_2)] = \begin{bmatrix} A_2 + M_2.(\frac{e^2}{4} + \left(L + \frac{D}{2}\right)^2) & M_2.\frac{H.L}{4} & M_2.\frac{H.e}{4} \\ M_2.\frac{H.L}{4} & A_2 + M_2.\frac{H^2 + e^2}{4} & -M_2.\left(L + \frac{D}{2}\right).\frac{e}{2} \\ M_2.\frac{H.e}{4} & -M_2.\left(L + \frac{D}{2}\right).\frac{e}{2} & A_2 + M_2.(\frac{H^2}{4} + \left(L + \frac{D}{2}\right)^2) \end{bmatrix}_{O,(\vec{x},\vec{y},\vec{z})}$$

$$[I_O(S_3)] = \begin{bmatrix} A_3 + M_3.(\frac{e^2}{4} + a^2) & M_3.\frac{H.a}{2} & M_3.\frac{H.e}{4} \\ M_3.\frac{H.a}{2} & A_3 + M_3.\frac{H^2 + e^2}{4} & -M_3.\frac{a.e}{2} \\ M_3.\frac{H.e}{4} & -M_3.\frac{a.e}{2} & C_3 + M_3.(\frac{H^2}{4} + a^2) \end{bmatrix}_{O,(\vec{x},\vec{y},\vec{z})}$$

$$[I_O(S_3')] = \begin{bmatrix} A_3 + M_3.(\frac{e^2}{4} + (a+b)^2) & M_3.\frac{H.(a+b)}{2} & M_3.\frac{H.e}{4} \\ M_3.\frac{H.(a+b)}{2} & A_3 + M_3.\frac{H^2+e^2}{4} & -M_3.\frac{(a+b).e}{2} \\ M_3.\frac{H.e}{4} & -M_3.\frac{(a+b).e}{2} & C_3 + M_3.(\frac{H^2}{4} + (a+b)^2) \end{bmatrix}_{O,(\vec{x},\vec{y},\vec{z})}$$

$$[I_O(S)] = [I_O(S_1)] + [I_O(S_2)] - [I_O(S_3)] - [I_O(S_3')]$$

$$[I_O(S)] = \begin{bmatrix} A_1 + A_2 - 2.A_3 + (M_1+M_2+M_3).\frac{e^2}{4} + M_1.\frac{L^2}{4} + M_2.\left(L+\frac{D}{2}\right)^2 - M_3.(a^2+(a+b)^2) & M_1.\frac{H.L}{4} + M_2.\frac{H}{2}.\left(L+\frac{D}{2}\right) - M_3.(a.H + \frac{b.H}{2}) & (M_1+M_2-2.M_3).\frac{H.e}{4} \\ M_1.\frac{H.L}{4} + M_2.\frac{H}{2}.\left(L+\frac{D}{2}\right) - M_3.(a.H + \frac{b.H}{2}) & B_1 + A_2 - 2.A_3 + (M_1+M_2+M_3).(\frac{H^2+e^2}{4}) & -M_1.\frac{L.e}{4} + M_2.\left(L+\frac{D}{2}\right).\frac{e}{2} - M_3.e.(a+\frac{b}{2}) \\ (M_1+M_2-2.M_3).\frac{H.e}{4} & -M_1.\frac{L.e}{4} + M_2.\left(L+\frac{D}{2}\right).\frac{e}{2} - M_3.e.(a+\frac{b}{2}) & C_1 + A_2 - 2.C_3 + M_1.\left(\frac{H^2+L^2}{4}\right) + M_2.\left(\frac{H^2}{4} + \left(L+\frac{D}{2}\right)^2\right) - M_3.(\frac{H^2}{2} + a^2 + (a+b)^2)) \end{bmatrix}_{O,(\vec{x},\vec{y},\vec{z})}$$

Q4 (Bonus) :

S_1 :

$$\begin{cases} A_1 = M_1.\frac{e^2+L^2}{12} \\ B_1 = M_1.\frac{e^2+H^2}{12} \\ C_1 = M_1.\frac{H^2+L^2}{12} \end{cases}$$

S_2 :

$$A_2 = M_2.\frac{D^2}{6}$$

S_3 :

$$\begin{cases} A_3 = M_3.(\frac{d^2}{16} + \frac{e^2}{16}) \\ C_3 = M_3.\frac{d^2}{8} \end{cases}$$