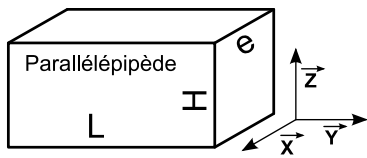


## TD – Matrices d’inertie / Palpeur MMT

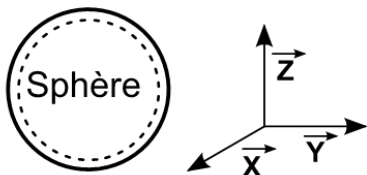
### POINT METHODE :

- Forme des matrices d’inertie (Q1) :



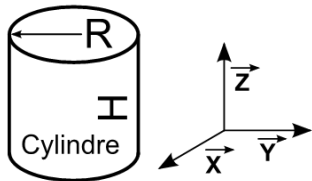
Deux Plans de symétrie  $(Q, \vec{x}, \vec{y})$  et  $(Q, \vec{x}, \vec{z})$  :

$$D=0 \text{ et } E=0 \text{ et } F=0 \rightarrow \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}_{Qxyz}$$



Un centre de symétrie sphérique (G forcément) :

$$A=B=C \text{ et } D=0 \text{ et } E=0 \text{ et } F=0 \text{ et } A+B+C=3A \rightarrow \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix}_Q$$



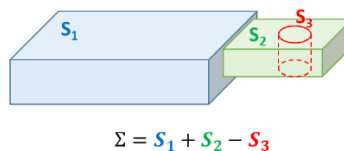
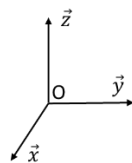
Un axe de Révolution  $(Q, \vec{z})$  :

$$A = B \text{ et } D=0 \text{ et } E=0 \text{ et } F=0 \text{ et } A+B=2A \rightarrow \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{bmatrix}_{Qz}$$

- Centre de gravité d’un ensemble de solides (Q2) :

$$\left(\sum_{i=1}^n m_i\right) \overrightarrow{OG} = \overrightarrow{MOG} = \sum_{i=1}^n m_i \overrightarrow{OG}_i \quad \text{ou} \quad \sum_{i=1}^n m_i \overrightarrow{GG}_i = \vec{0}$$

- Somme de matrices d’inertie (Q3) :



$$[I_O(\Sigma)]_R = [I_O(S_1)]_R + [I_O(S_2)]_R - [I_O(S_3)]_R$$

- Théorème de Huygens (Q3) :

$$[I_P(S)] = [I_G(S)] + m \begin{bmatrix} b^2 + c^2 & -ab & -ac \\ -ab & a^2 + c^2 & -bc \\ -ac & -bc & a^2 + b^2 \end{bmatrix} \quad \text{Avec } \overrightarrow{PG} = a \vec{x} + b \vec{y} + c \vec{z}$$

ELEMENTS DE CORRECTION :

**Q1 :**

$$G_1 = \begin{pmatrix} -\frac{H}{2} \\ L \\ \frac{e}{2} \end{pmatrix}_{(\vec{x}, \vec{y}, \vec{z})} \quad \text{et } [I_{G_1}(S_1)] = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & B_1 & 0 \\ 0 & 0 & C_1 \end{bmatrix}_{G_1, (\vec{x}, \vec{y}, \vec{z})}$$

$$G_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{(\vec{x}, \vec{y}, \vec{z})} \quad \text{et } [I_{G_2}(S_2)] = \begin{bmatrix} A_2 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_2 \end{bmatrix}_{G_2, (\vec{x}, \vec{y}, \vec{z})}$$

$$G_3 = \begin{pmatrix} 0 \\ 0 \\ \frac{e}{2} \end{pmatrix}_{(\vec{x}, \vec{y}, \vec{z})} \quad \text{et } [I_{G_3}(S_3)] = \begin{bmatrix} A_3 & 0 & 0 \\ 0 & A_3 & 0 \\ 0 & 0 & C_3 \end{bmatrix}_{G_3, (\vec{x}, \vec{y}, \vec{z})}$$

**Q2 :**

$$\vec{OG} = -\frac{H}{2} \cdot \vec{x} + \frac{M_1 \cdot \frac{L}{2} + M_2 \cdot \left(L + \frac{D}{2}\right) - M_3 \cdot (2 \cdot a + b)}{M_1 + M_2 + M_3} \cdot \vec{y} + \frac{e}{2} \cdot \vec{z}$$

**Q3 :**

$$[I_O(S_1)] = \begin{bmatrix} A_1 + M_1 \cdot \frac{L^2 + e^2}{4} & M_1 \cdot \frac{H \cdot L}{4} & M_1 \cdot \frac{H \cdot e}{4} \\ M_1 \cdot \frac{H \cdot L}{4} & B_1 + M_1 \cdot \frac{H^2 + e^2}{4} & -M_1 \cdot \frac{L \cdot e}{4} \\ M_1 \cdot \frac{H \cdot e}{4} & -M_1 \cdot \frac{L \cdot e}{4} & C_1 + M_1 \cdot \frac{L^2 + H^2}{4} \end{bmatrix}_{O, (\vec{x}, \vec{y}, \vec{z})}$$

$$[I_O(S_2)] = \begin{bmatrix} A_2 + M_2 \cdot \left(\frac{e^2}{4} + \left(L + \frac{D}{2}\right)^2\right) & M_2 \cdot \frac{H \cdot L}{4} & M_2 \cdot \frac{H \cdot e}{4} \\ M_2 \cdot \frac{H \cdot L}{4} & A_2 + M_2 \cdot \frac{H^2 + e^2}{4} & -M_2 \cdot \left(L + \frac{D}{2}\right) \cdot \frac{e}{2} \\ M_2 \cdot \frac{H \cdot e}{4} & -M_2 \cdot \left(L + \frac{D}{2}\right) \cdot \frac{e}{2} & A_2 + M_2 \cdot \left(\frac{H^2}{4} + \left(L + \frac{D}{2}\right)^2\right) \end{bmatrix}_{O, (\vec{x}, \vec{y}, \vec{z})}$$

$$[I_O(S_3)] = \begin{bmatrix} A_3 + M_3 \cdot \left(\frac{e^2}{4} + a^2\right) & M_3 \cdot \frac{H \cdot a}{2} & M_3 \cdot \frac{H \cdot e}{4} \\ M_3 \cdot \frac{H \cdot a}{2} & A_3 + M_3 \cdot \frac{H^2 + e^2}{4} & -M_3 \cdot \frac{a \cdot e}{2} \\ M_3 \cdot \frac{H \cdot e}{4} & -M_3 \cdot \frac{a \cdot e}{2} & C_3 + M_3 \cdot \left(\frac{H^2}{4} + a^2\right) \end{bmatrix}_{O, (\vec{x}, \vec{y}, \vec{z})}$$

$$[I_O(S_3')] = \begin{bmatrix} A_3 + M_3 \cdot \left(\frac{e^2}{4} + (a+b)^2\right) & M_3 \cdot \frac{H \cdot (a+b)}{2} & M_3 \cdot \frac{H \cdot e}{4} \\ M_3 \cdot \frac{H \cdot (a+b)}{2} & A_3 + M_3 \cdot \frac{H^2 + e^2}{4} & -M_3 \cdot \frac{(a+b) \cdot e}{2} \\ M_3 \cdot \frac{H \cdot e}{4} & -M_3 \cdot \frac{(a+b) \cdot e}{2} & C_3 + M_3 \cdot \left(\frac{H^2}{4} + (a+b)^2\right) \end{bmatrix}_{O,(\vec{x},\vec{y},\vec{z})}$$

$$[I_O(S)] = [I_O(S_1)] + [I_O(S_2)] - [I_O(S_3)] - [I_O(S_3')]$$

$$[I_O(S)] = \begin{bmatrix} A_1 + A_2 - 2 \cdot A_3 + (M_1 + M_2 - 2 \cdot M_3) \cdot \frac{e^2}{4} + M_1 \cdot \frac{L^2}{4} + M_2 \cdot \left(L + \frac{D}{2}\right)^2 - M_3 \cdot (a^2 + (a+b)^2) & M_1 \cdot \frac{H \cdot L}{4} + M_2 \cdot \frac{H}{2} \cdot \left(L + \frac{D}{2}\right) - M_3 \cdot \left(a \cdot H + \frac{b \cdot H}{2}\right) & (M_1 + M_2 - 2 \cdot M_3) \cdot \frac{H \cdot e}{4} \\ M_1 \cdot \frac{H \cdot L}{4} + M_2 \cdot \frac{H}{2} \cdot \left(L + \frac{D}{2}\right) - M_3 \cdot \left(a \cdot H + \frac{b \cdot H}{2}\right) & B_1 + A_2 - 2 \cdot A_3 + (M_1 + M_2 - 2 \cdot M_3) \cdot \left(\frac{H^2 + e^2}{4}\right) & -M_1 \cdot \frac{L \cdot e}{4} + M_2 \cdot \left(L + \frac{D}{2}\right) \cdot \frac{e}{2} - M_3 \cdot e \cdot \left(a + \frac{b}{2}\right) \\ (M_1 + M_2 - 2 \cdot M_3) \cdot \frac{H \cdot e}{4} & -M_1 \cdot \frac{L \cdot e}{4} + M_2 \cdot \left(L + \frac{D}{2}\right) \cdot \frac{e}{2} - M_3 \cdot e \cdot \left(a + \frac{b}{2}\right) & C_1 + A_2 - 2 \cdot C_3 + M_1 \cdot \left(\frac{H^2 + L^2}{4}\right) + M_2 \cdot \left(\frac{H^2}{4} + \left(L + \frac{D}{2}\right)^2\right) - 2 \cdot M_3 \cdot \left(\frac{H^2}{4} + a^2 + (a+b)^2\right) \end{bmatrix}_{O,(\vec{x},\vec{y},\vec{z})}$$

**Q4 (Bonus) :**

$S_1 :$

$$\begin{cases} A_1 = M_1 \cdot \frac{e^2 + L^2}{12} \\ B_1 = M_1 \cdot \frac{e^2 + H^2}{12} \\ C_1 = M_1 \cdot \frac{H^2 + L^2}{12} \end{cases}$$

$S_2 :$

$$A_2 = M_2 \cdot \frac{D^2}{6}$$

$S_3 :$

$$\begin{cases} A_3 = M_3 \cdot \left(\frac{d^2}{16} + \frac{e^2}{16}\right) \\ C_3 = M_3 \cdot \frac{d^2}{8} \end{cases}$$